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LEARNING & CREATIVITY PLAN (L&C PLAN): Symmetry

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1. Overview

Title	Symmetry
Driving Question or Topic	Identify symmetry in nature and explore its mathematical background and theory
Ages, Grades, …	Ages: 13-15 7-9 grades
Duration, Timeline, Activities	4 learning units, 8 hours 4*90 minutes 4 activities
Curriculum Alignment	Mathematics, Arts, Physics, Sciences
Contributors, Partners	
Abstract - Synopsis	Students are encouraged to find examples of symmetry in nature. Examples can be found in living organisms, architecture, music, physics and chemistry. Inductive reasoning is used to derive theoretical foundations of symmetry in mathematics.
References, Acknowledgements	

2. STEAME Framework*

Teachers' Cooperation	Teacher 1 (T1) – mathematics A3: Collects natural and abstract models found and leads discussions towards identifying their mathematical models and deriving the true nature of symmetry. A4: Explains mathematical models and explores their symmetries in a formal way. Teacher 2 (T2) – sciences: physics, chemistry A1: Leads discussion in class or during a field visit of surrounding symmetrical objects. Once line symmetry is identified, encourages discussion of space symmetry. Explains appearances of crystals in nature and human made. Teacher 3 (T3) – arts or language A2: Discusses symmetries in literature, music, architecture, painting. Provides examples and non-examples. The teachers can work together in arbitrary configuration or can teach their parts individually. Activities A1 and A2 can be carried out in arbitrary order.
STEAME in Life (SiL) Organization	Working with examples, field visit: architecture, painting gallery, jewelry shop. Making explicit measurements of natural objects.
Action Plan Formulation	There are 4 activities involved. A1 and A2 are interchangeable. A3 and A4 are interchangeable as well. They serve as the sum-up of the plan.

* *under development the final elements of the framework*

3. Objectives and Methodologies

Learning Goals and Objectives	Completing the L&C Plan, students will • Understand the meaning of symmetry;

	• Distinguish and name various basic types of symmetry; • Name symmetric patterns present in the nature; • Comprehend symmetry role in arts and literature; • Be able to present results of the project in written and oral form.
Learning Outcomes and expected Results	Students will • Remember definition of symmetry and basics of their classification; • Understand role of symmetry in everyday live, arts and physical systems. • Be able to apply gained information towards versification of the presence of symmetry in new situations and objects. • Be able to tell difference between less and more symmetry; • Be able to generalize gained knowledge to other concepts, e.g. similarity.
Prior Knowledge and Prerequisites	This L&C Plan does not require any prior knowledge.
Motivation, Methodology, Strategies, Scaffolds	Students shall comprehend real live roots of an important mathematical concept. They will be confronted with inductive method in science which starts with collecting data and seeks for an abstract generalization, theory. Students may discuss issues such as: is the symmetry understanding learned or are we born with this concept. Why is the linear symmetry the most common, etc. The goals of the L&C Plan can be achieved with an inquiry-based or project-based approach. Supervision and light guidance of discussions is required throughout the project.

4. Preparation and Means

Preparation, Space Setting, *Troubleshooting Tips*	The L&C should start with identifications of symmetry in the real world. This can be achieved in the classroom and outside of it. The discussion can be triggered by using a mirror. A mirror can be replaced by a smartphone with a camera facing its user switched on. One can multiply the effect facing two cameras and discuss why facing three cameras is not possible. Axial and planar symmetries are the easiest to identify. Then one can search for more complex symmetries and objects having high number of symmetries (even infinitely many, e.g. ideal mathematical objects such as a circle, a line, a torus etc.). Theoretical, mathematical framework should be taught in the classroom.
Resources, Tools, Material, Attachments, Equipment	It is possible to use objects naturally creating symmetry, e.g., mirrors, cameras. Pictures of symmetric object might be helpful. One can consider natural crystals and go to platonic solids and their symmetries. Possible diversion here is to let students grow their own crystals at home. There are instructions available online, which explain how this can be done effectively, e.g., https://www.youtube.com/watch?v=kKLga-8IMiY Inspirations for symmetry in the literature can be found here: https://www.sciencedirect.com/science/article/pii/0898122186901513 Symmetries in music are considered here: http://www.mi.sanu.ac.rs/vismath/visbook/apagyi/index.html Nice introduction to symmetry in the architecture, already organized in form of a lesson is available here: http://www.mi.sanu.ac.rs/vismath/visbook/apagyi/index.html Additional material is presented here: https://www.mi.sanu.ac.rs/vismath/kim/index.html
Safety and Health	There are no particular safety measures required by this L&C Plan.

5. Implementation

Instructional Activities, Procedures, Reflections	This L&C Plan requires 4 units, 90 minutes each. It can begin either with T2 or T3 explaining symmetry in the real world or in the world of arts. T2 seeks with students examples of symmetry in the real world, collects them and classifies types of encountered symmetry. (2 hours) T3 provides initial examples of symmetry in arts. Most likely the students will come up with examples in painting, sculpture and architecture first. It is less obvious to come up with examples in the literature and the music and this might require some instruction. (2 hours) T1 provides theoretical, mathematical background for the concept of symmetry. More importantly, T1 explains how examples from the real life are abstracted to the ideal world of mathematics. This provides opportunity to recall similar process for numbers (e.g. passing from concrete 2 cars to the abstract number 2). Some classification of planar and/or special symmetries should finish the course. (4 hours)
Assessment - Evaluation	Each activity can be accompanied by quick tests checking the comprehension of discussed concepts and examples. A possible way to test is to use a hand-out with a picture exhibiting a number of symmetries and asking to identify them. A more funny way to test, which increases considerably, the focus of students is by peer questions with immediate presentation of results using , e.g., https://pingo.coactum.de/ query tool. If prepared well, one can the opportunity

	to discuss the Gauss curve and its symmetry https://www.sciencedirect.com/topics/engineering/gaussian-curve At the end students should complete a short (5-10 minutes) multiple choice test, which can be carried out using electronic devices.
Presentation - Reporting - Sharing	Students might be required to collect examples of symmetric objects. If they come from real world they can create an exhibition, if they have only pictures of them, they can create a gallery. It is important that some examples come in the form hands on rather than digital.
Extensions - Other Information	A more general concept than symmetry is similarity. This is, in a sense, symmetry with a scale. Or just a scale. Again multiple examples are present in the real world and can motivate passage to abstract approaches.

STEAME Prototype/Guide for Learning & Creativity Approach
Action Plan Formulation

Major steps in the STEAME learning approach:

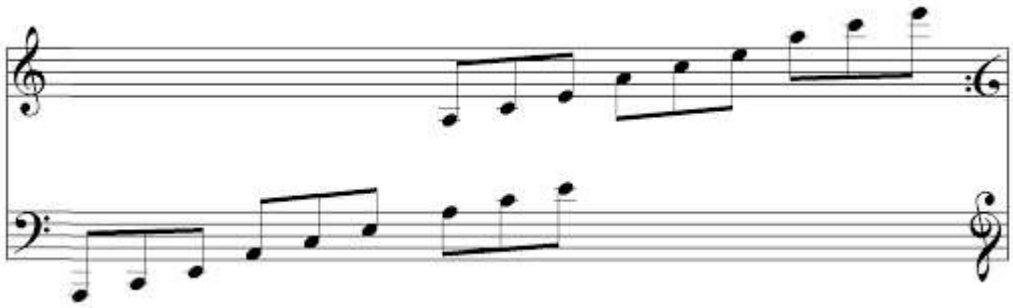
STAGE I: Preparation by one or more teachers

1. Formulating initial thoughts on the thematic sectors/areas to be covered
    - Symmetry and its presence in world and science;
    - Collecting initial examples;
    - Discussing approach to the problem in the presence of students.
2. Engaging the world of the wider environment / work / business / parents / society / environment/ ethics
    - Universality of abstract concepts. Inclusive character of science. Is there a good-evil symmetry?
3. Target Age Group of Students - Associating with the Official Curriculum - Setting Goals and Objectives
    - Can be worked out with students of grades 7-9.
    - Association to regular Curriculum should be discussed in any specific given case.
    - For Goals and Objectives see part 3 of the L&C.
4. Organization of the tasks of the parties involved - Designation of Coordinator - Workplaces etc.
    - Teacher 2 of sciences (physics, chemistry, geography, biology)
      Seeks with students examples of symmetry in the real world.
      This can be supported by mobile devices use. Can visit laboratory to inspect e.g. structure of crystals. Can visit architecture bureau. Can go out and seek symmetries in trees, everyday use devices etc. Can be taught also in the classroom without hands-on approach.
    - Teacher 3 of arts or language
      Presents initial examples of symmetries in arts and literature. Encourages students to come up with their own examples. Evaluates examples presented. Encourages discussion but tries not to lead it. Can be taught in a classroom, art gallery, exhibition etc.
    - Teacher 1 of mathematics
      Initiates discussion of underlying principles of symmetry encountered in lessons with T2 and T3. Provides guidelines for the principle of abstraction. This can be illustrated with examples such as the concept of a number, the concept of a segment, triangle, circle. Makes tights to philosophical ideas of Plato and his followers.

STAGE II: Action Plan Formulation (Steps 1-18)

<u>Preparation (by teachers)</u>

1. Relation to the Real World – Reflection
    - Real world is full of symmetries. Identify examples. Pictures below present two objects exhibiting symmetry: Pentagon building and a music passage.



2. Incentive – Motivation
    - Explore basic concepts of the universe.
3. Formulation of a problem (possibly in stages or phases) resulting from the above
    - Based on a large series of collected examples, the idea is to pass to the abstract concept of symmetry.
      Apart of working with this specific concept, students should be exposed to the process of abstraction.

Development (by students) – Guidance & Evaluation (in 9-11, by teachers)

4. Background Creation - Search / Gather Information
    - Students create a large collection of symmetries ranging from objects in everyday life. Possible examples are: spoons, glasses, cloths. Passing to objects in wide understood arts, including architecture, painting, music, literature and coming to examples in sciences like crystals and galaxies.
5. Simplify the issue - Configure the problem with a limited number of requirements
    - Rather than simplify students should think towards properties joining all observed symmetries. In particular determine their number in specific cases.
6. Case Making - Designing - identifying materials for building / development / creation
    - Does not apply.
7. Construction - Workflow - Implementation of projects
    - Students can work in groups discussing and enhancing gathered sets of examples e.g. in the architecture and other rough groups.
8. Observation-Experimentation - Initial Conclusions
    - Students should identify what various categories of examples have in commen.

9. Documentation - Searching Thematic Areas (STEAME fields) related to the subject under study – Explanation based on Existing Theories and / or Empirical Results
    - Students should become aware of symmetry presence in engineering objects, in arts, in mathematics, sciences, arts and even in economy discussing concepts of debts and investments.
10. Gathering of results / information based on points 7, 8, 9
    - Results should be gathered and prepared for presentation.
11. First group presentation by students
    - Results of working groups are presented and discussed with peers.

Configuration & Results (by students) – Guidance & Evaluation (by teachers)

12. Configure mathematics or other STEAME models to describe / represent / illustrate the results
    - Collected examples can be presented e.g. in a form of a booklet with their depictions, a gallery, an internet collection etc.
13. Studying the results in 9 and drawing conclusions, using 12
    - Students are encouraged to draw conclusions on the abstract concept of symmetry.
14. Applications in Everyday Life - Suggestions for Developing 9 (Entrepreneurship - SIL Days)
    - Does not apply.

Review (by teachers)

15. Review the problem and review it under more demanding conditions
    - The concept of symmetry in mathematics is presented and analyzed. One can attempt to classify symmetries if the group works well. Symmetries focus on geometry but this should include graphs of functions and numbers.

Project Completion (by students) – Guidance& Evaluation (by teachers)

16. Repeat steps 5 through 11 with additional or new requirements as formulated in 15
17. Investigation - Case Studies - Expansion - New Theories - Testing New Conclusions
18. Presentation of Conclusions - Communication Tactics.
    - An output of the project should be a writing explaining what was done. It should mention some examples, explain the passage to the abstract concept and conclude with rigorous mathematical statements and formulations.

STAGE III: STEAME Actions and Cooperation in Creative Projects for school students

Title of STEAME Project : _____
Brief Description/Outline of Organizational Arrangements / Responsibilities for Action

STAGE	**Activities/Steps** Teacher 1(T1) Cooperation with T2 and student guidance	**Activities /Steps** **By Students** Age Group: ____	**Activities /Steps** Teacher 2 (T2) Cooperation with T1 and student guidance
A	Preparation of steps 1,2,3		Cooperation in step 3
B	Guidance in step 9	4,5,6,7,8,9,10	Support guidance in step 9
C	Creative Evaluation	11	Creative Evaluation
D	Guidance	12	Guidance

E	Guidance	13 (9+12)	Guidance
F	Organization (SIL) STEAME in Life	14 Meeting with Business representatives	Organization (SIL) STEAME in Life
G	Preparation of step 15		Cooperation in step 15
H	Guidance	16 (repetition 5-11)	Support Guidance
I	Guidance	17	Support Guidance
K	Creative Evaluation	18	Creative Evaluation