## Graph Theory and real life applications.

## Exercise 1:

a) Write down the degree of each vertex


| Vertex | Degree |
| :---: | :---: |
| A |  |
| B |  |
| C |  |
| D |  |
| E |  |
| F |  |

b) Write down the sum of the degrees of all vertices.

Exercise 2:
a) Can you draw a graph with the sum of degree of vertices to be an odd number?
b) Is it possible for 5 people to shake hand with exactly 3 of them?

Exercise 3:
a) In an octagon, what is the sum of its edges, if we draw all of its diagonal?
b) Write down the general formula for the number of sides of a closed polygon if we join all its vertices.

Exercise 4:
a) Which of the following graphs have a Eulerian path?
i)

ii)

iii)

b) For those graph that there exist a Eulerian path find it.

## Exercise 5:

Which of the following graphs have a Eulerian circuit?
i)

ii)

iii)


## Exercise 6:

Solve the route inspection problem for this network starting and finishing at A.


Information for the problem
Below is a list of all trains servicing the northeast, as well as the connecting cities.
Acela Express: Boston $\rightarrow$ New York $\rightarrow$ Philadelphia $\rightarrow$ Washington
Capital Limited: Pittsburgh $\rightarrow$ Cleveland $\rightarrow$ Waterloo $\rightarrow$ Chicago
Cardinal: Washington $\rightarrow$ Hinton $\rightarrow$ Cincinnati $\rightarrow$ Lafayette $\rightarrow$ Chicago
The Federal: Boston $\rightarrow$ New York $\rightarrow$ Philadelphia $\rightarrow$ Washington
Keystone: New York $\rightarrow$ Philadelphia $\rightarrow$ Harrisburg
Lake Shore Limited: Chicago $\rightarrow$ Cleveland $\rightarrow$ Albany $\rightarrow$ Boston or New York
Metroliner: New York $\rightarrow$ Philadelphia $\rightarrow$ Washington
Pennsylvanian: Pittsburgh $\rightarrow$ Philadelphia $\rightarrow$ New York
Three Rivers: New York $\rightarrow$ Youngstown $\rightarrow$ Chicago

Number of miles between any two cities

|  | Albany | Boston | New <br> York | Philly | DC |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Albany | 0 | 144 | 138 | 216 | 342 | 532 | 646 | 958 | 1208 | 310 |
| Boston | 144 | 0 | 282 | 360 | 486 | 676 | 790 | 1102 | 1352 | 376 |
| New <br> York | 138 | 282 | 0 | 78 | 204 | 394 | 508 | 820 | 1070 | 172 |
| Philly | 216 | 360 | 78 | 0 | 126 | 316 | 430 | 742 | 992 | 94 |
| Wash DC | 342 | 486 | 204 | 126 | 0 | 190 | 304 | 616 | 866 | 220 |
| Pitts | 532 | 676 | 394 | 316 | 190 | 0 | 114 | 426 | 676 | 410 |
| Cleveland | 646 | 790 | 508 | 430 | 304 | 114 | 0 | 312 | 562 | 524 |
| Chicago | 958 | 1102 | 820 | 742 | 616 | 426 | 312 | 0 | 250 | 836 |
| Cincy | 1203 | 1352 | 1070 | 992 | 866 | 676 | 562 | 250 | 0 | 1086 |
| Harris | 310 | 376 | 172 | 94 | 220 | 410 | 524 | 836 | 1086 | 0 |

Exercise:
a) Connect the cities using graphs
b) Chose a path that can optimized the trains routes for connected cities.
c) Identify the most central cities based on their distances and place them in rank position.
(hint the city with the smallest distance to all the rest will be the most central)
d) Chose a path that can optimized the trains routes with respect of their distances.

